**Round Robin Calibration Model Document**

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Goal: To determine if low fidelity model can accurately capture experiment with calibration alone. Then apply a nonlinear inadequacy operator to see if it can match experiment better than the low fidelity model.

**Low Fidelity Model**

**High Fidelity Model**

Where,

**Constants for Calibration Scenario**

|  |  |
| --- | --- |
| k | 25788 N/m |
| b | 18.206 Ns/m |
| g0 | 26 Ns/m |
| kf | 722 N/m |
| bf | 0.176 Ns/m |

To prevent negative values for calibration but to maintain a normal distribution a log normal distribution was used for the calibration of 5 constants. A uniform distribution with limits between -0.5 and 0.5 was used for the initial positions and velocities for the calibration. A MAP estimate was performed as a starting point for MCMC chains.

**Priors for Calibration of Low Fidelity Model**

|  |  |  |
| --- | --- | --- |
| Constant | Mean | Standard Deviation |
| k | 25788 | 0.04 |
| b | 18.206 | 0.05 |
| g0 | 26 | 0.02 |
| kf | 722 | 0.07 |
| bf | 0.176 | 0.02 |

**Priors for Calibration of High Fidelity Model**

Priors are the same as the low fidelity model with the addition of the two constants for the inadequacy operator.

|  |  |  |
| --- | --- | --- |
| Constant | Mean | Standard Deviation |
| k1 | 500 | 0.05 |
| k2 | 20000 | 0.03 |

**A graph with a red line

Description automatically generatedResults for MAP Estimate and MCMC Calibration for Low Fidelity Model**

|  |  |
| --- | --- |
| k | 26830 N/m |
| b | 9.6 Ns/m |
| g0 | 21.4 Ns/m |
| kf | 877 N/m |
| bf | 0.175 Ns/m |
| x1 | 0.00026 |
| x2 | 0.00045 |
| v1 | 0.0091 |
| v2 | 0.0095 |

A graph of a graph

Description automatically generated

Table shows starting values from MAP estimate for MCMC(Markov Chain Monte Carlo) Calibration. Calibrated graph is plotted with 50 and 95% confidence intervals and a validation fraction of 0.00399 was observed. Meaning that about 99.6% of calibrated data was within a 95% confidence interval of experimental data.

**MCMC Chains for Low Fidelity**

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Description automatically generated

Chains for calibration were primarily stable. However, if I had more time I would have done more iterations. Chain for the constant k could be more stable. This is with running for 300,000 steps. If steps were increased to around 400,000-500,000 I believe the constant k would have mixed more. Another approach would be to try increasing the number of parallel chains.

**Validation Scenarios for Low Fidelity**

To determine if the calibrated low fidelity model can accurately capture experiment did a few validation scenarios. I took data from my chains sampled at every 200th step and plotted the 95th percentile and 50th percentile trajectories compared to experiment.

|  |  |  |
| --- | --- | --- |
| Mass | 0.9249 | 1.0613 |
| Validation Fraction | 0.0869 | 0.0618 |

A graph of a graph

Description automatically generatedA graph with a line graph

Description automatically generated with medium confidence

mass=0.9249 mass=1.0613

The mass of 0.9249 and 1.0613 with calibrated data match experiment within an approximately 91% and 94% range based on validation of a 95% confidence interval respectively. Larger mass matched experiment better with the calibrated constants of the low fidelity model.

**High Fidelity Model**

To determine if an inadequacy operator to estimate the nonlinear stiffness. In a spring systems often, there is stiffness in the spring which is not accounted for with linear equations. Thus, they cannot accurately capture variations in stiffness and need a nonlinear correction. In low fidelity model is linear so by adding a nonlinear inadequacy operator can better describe the system. The added constants are k1 and k2 such that is added as an inadequacy operator to the low fidelity model.

**Results for MAP Estimate and MCMC Calibration for High Fidelity Model**

|  |  |
| --- | --- |
| k | 26680 N/m |
| b | 9.6 Ns/m |
| g0 | 21.8 Ns/m |
| kf | 916N/m |
| bf | 0.179 Ns/m |
| k1 | 504 N/m2 |
| k2 | 20492 N/m3 |
| x1 | 0.0003 |
| x2 | 0.00049 |
| v1 | 0.0068 |
| v2 | 0.0065 |

A graph with a red line

Description automatically generated

A graph with a line graph

Description automatically generated with medium confidence

Table shows results from MAP estimate, or the starting values for the MCMC calibration. From validation a fraction of 0.00199. Showing slightly better fit than the calibrated low fidelity model with about 99.8% of data falling within experiment for a 95% confidence interval.

**MCMC Chains for High Fidelity Model**

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Description automatically generated with medium confidence

Chains primarily look good, the only chain that needs more iterations is chain for the second spring constant. This was run for 200,000 steps. If I were to run again, I would do 300,000 to 400,000 steps or increasing parallel chains over 5. Chains were sampled every 200 steps.

**Validation Scenarios for High Fidelity**

To determine if the calibrated high fidelity model can accurately capture experiment did a few validation scenarios. I took data from my chains sampled at every 200th step and plotted the 95th percentile and 50th percentile trajectories compared to experiment.

|  |  |  |
| --- | --- | --- |
| Mass | 0.9249 | 1.0613 |
| Validation Fraction | 0.0546 | 0.0937 |

A graph of a graph

Description automatically generatedA graph of a graph

Description automatically generated

mass=0.9249 mass=1.0613

For the masses of 0.9249 and 1.0613 based on validation fraction approximately 94% and 90% of data respectively fall within the 95% confidence interval. Overall validation shows results from calibrated low fidelity and calibrated high fidelity yield comparable results.

**Conclusion**

Overall calibrated high fidelity model and calibrated low fidelity model can accurately capture experiment with at least approximately 90% of the data falling in the range of experiment within a 95% confidence interval. High fidelity model did not perform substantially better than low fidelity. Thus, with significant calibration low fidelity can capture experiment well.